Kennedy Baptist College

WAEP Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section One: Calculator-free

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	10	10	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

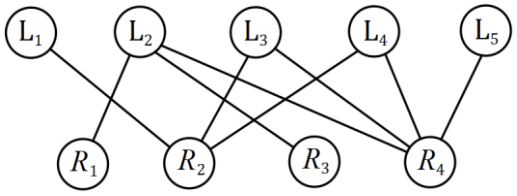
Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Five university lecturers (L_1, L_2, L_3, L_4 and L_5) have been allocated four rooms (R_1, R_2, R_3 and R_4) to teach in. Only one lecturer will teach in a room at any time. Because some of the lecturers require specialist equipment, not all the rooms can be used by all the lecturers, as shown in the graph below.



(a) What is the name of such a graph shown above, where the vertices can be split into two groups so that each edge joins a vertex from one group to a vertex in the other group? (1 mark)

Solution
Bipartite
Specific behaviours
✓ correct name

(b) How many lecturers can use room R_4? (1 mark)

Solution
4 lecturers.
Specific behaviours
✓ correct number

(c) How many rooms can lecturer L_2 use? (1 mark)

Solution
3 rooms.
Specific behaviours
✓ correct number

(d) Briefly explain whether

(i) all five lecturers can teach at the same time? (1 mark)

Solution
No - more lecturers than rooms.
Specific behaviours
✓ no, with valid reason

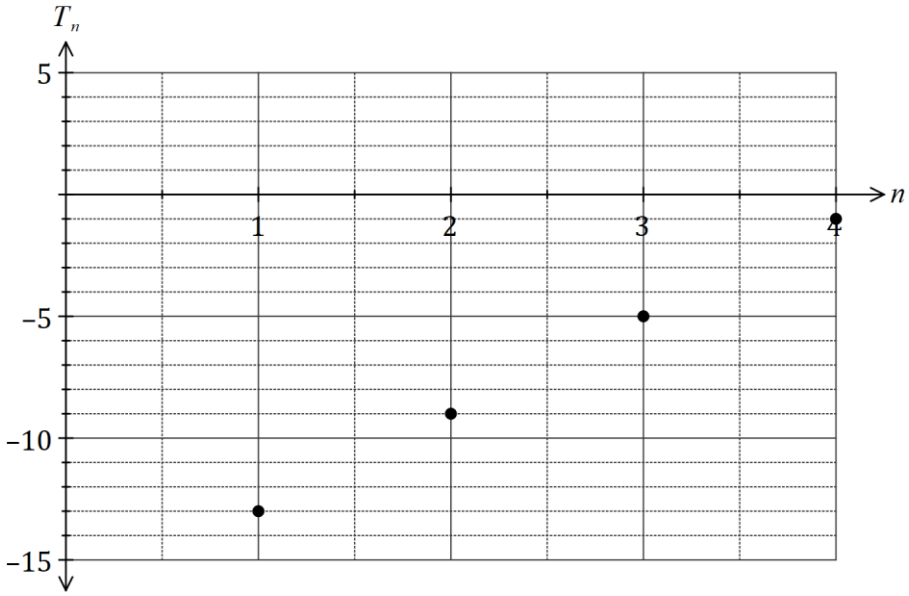
(ii) all four rooms could be in use at the same time? (1 mark)

Solution
No - R_1 and R_3 only used by L_2.
Specific behaviours
✓ no, with valid reason

See next page

Question 2 **(4 marks)**

The first four terms of an arithmetic sequence are shown on the graph below.



(a) Deduce a rule for the n^{th} term of this sequence. (2 marks)

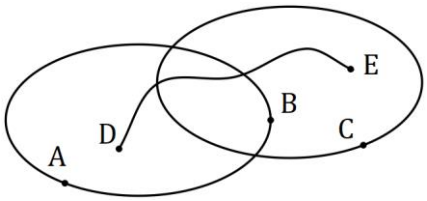
Solution
$T_n = -13 + (n-1)(4)$ $= 4n - 17$
Specific behaviours
✓ identifies common difference ✓ rule (accept any correct form)

(b) Given that the k^{th} term of this sequence is 399, determine the value of k. (2 marks)

Solution
$-13 + (k-1)(4) = 399$ $(k-1)(4) = 412$ $k - 1 = \frac{412}{4} = 103$ $k = 104$
Specific behaviours
✓ uses equation from (a) ✓ solves for k

Question 3 **(7 marks)**

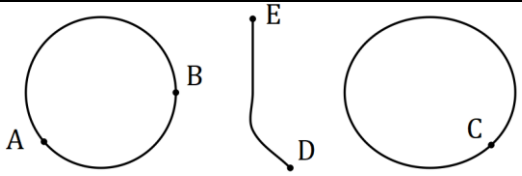
Graph G_1 is shown below with vertices A, B, C, D and E.



(a) In graph theory, a planar graph is a graph that can be drawn in the plane. Describe how to draw the edges of such a graph to clearly show that it is planar. (1 mark)

Solution
Draw so that no two edges cross, or, draw so that edges only meet at vertices.
Specific behaviours
✓ description

(b) Redraw graph G_1 to clearly show that it is planar. (2 marks)

Solution

Specific behaviours
✓ topological equivalent, labelled vertices ✓ no two edges cross

(c) State the number of loops graph G_1 contains. (1 mark)

Solution
1 loop
Specific behaviours
✓ number of loops

(d) Show that Euler's formula does not apply to graph G_1. (3 marks)

Solution
$f = 3, v = 5, e = 4$ $f + v - e = 3 + 5 - 4 = 4$ Hence Euler's formula does not apply as result should be 2, not 4.
Specific behaviours
✓ correct f, v, e ✓ substitutes into formula ✓ indicates that result is not 2

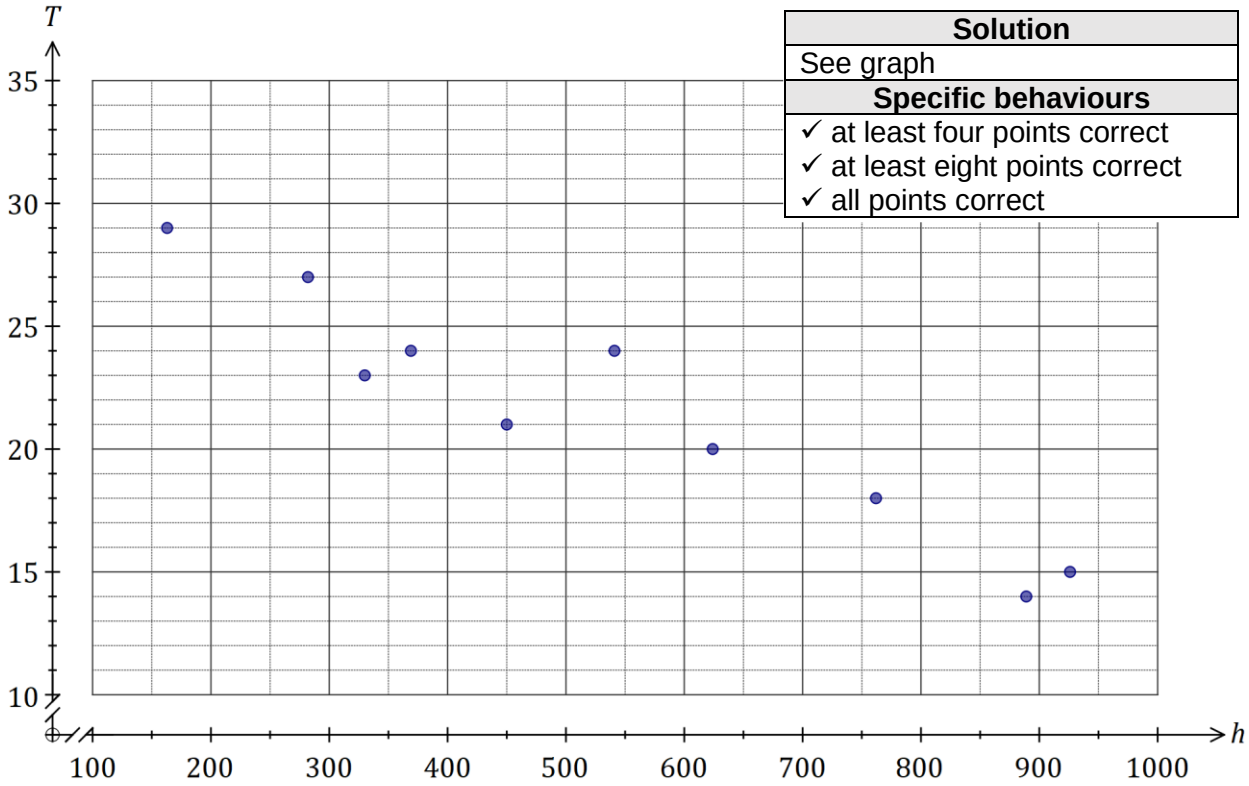
See next page

Question 4 **(8 marks)**

The average maximum temperature, T °C, was recorded for ten weather stations, together with the altitude of the station, h metres. The data is shown in the table below.

Altitude, h	163	282	330	369	450	541	624	762	889	926
Temperature, T	29	27	23	24	21	24	20	18	14	15

(a) Construct a scatterplot on the axes below that can be used to identify whether an association exists between altitude and temperature. (3 marks)



Solution
See graph
Specific behaviours
✓ at least four points correct
✓ at least eight points correct
✓ all points correct

(b) Describe the features of the scatterplot that indicate a strong, negative and linear association exists between altitude and temperature. (2 marks)

Solution
Strong & linear: points lie close to a straight line Negative: as altitude increases, the temperature decreases
Specific behaviours
✓ strong & linear reason ✓ negative reason

(c) Estimate a value for

(i) the temperature at an altitude of 500 metres. (1 mark)

Solution
Accept $21 \leq T \leq 23$
Specific behaviours
✓ within given range

(ii) the value of the correlation coefficient between the two variables. (2 marks)

Solution
Accept $-0.99 \leq r \leq -0.80$
Specific behaviours
✓ negative value ✓ within given range

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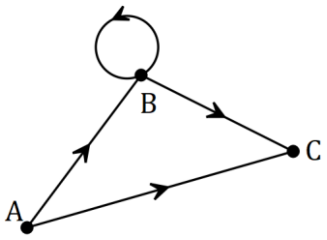
Question 5 **(7 marks)**

A digraph is shown below.



(a) State, with justification, whether the digraph contains

(i) a walk of length 6. (2 marks)

Solution
Yes. Start at B and travel around the loop 6 times.
Specific behaviours
✓ states yes ✓ lists example walk or other justification

(ii) a Hamiltonian path. (2 marks)

Solution
Yes. The path is ABC.
Specific behaviours
✓ states yes ✓ lists path or other justification

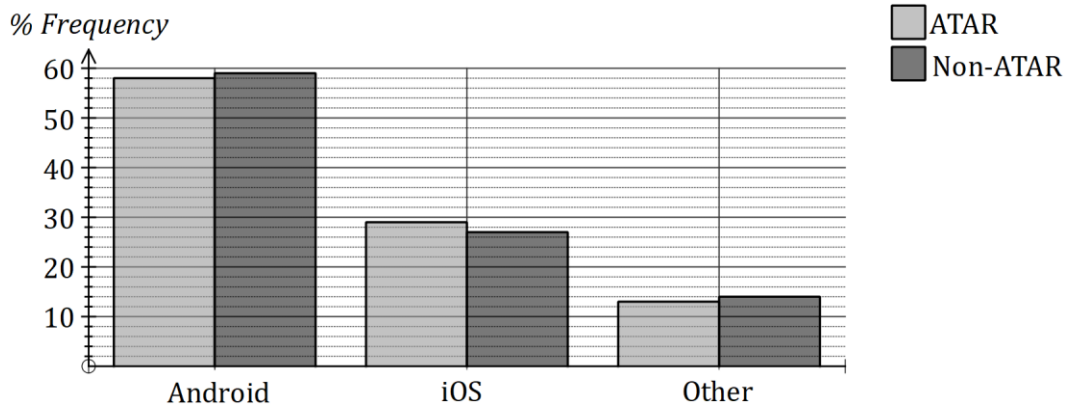
(b) Using column and row headings in the order $A - B - C$, construct the adjacency matrix M for the digraph and explain what the number in the first row and third column of matrix M^3 represents. (3 marks)

Solution
$M = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $M^3_{1,3}$ is the number of ways to travel from A to C along 3 edges.
Specific behaviours
✓ at least two rows of M correct ✓ M correct ✓ states start, finish vertices and edges used

See next page

Question 6 **(6 marks)**

Mobile phone users who responded to a survey were asked which type of operating system their current phone used (Android, iOS or other) and whether they followed an ATAR or non-ATAR pathway at school. A breakdown of the results is shown in the graph below.



(a) What percentage of the respondents who followed a non-ATAR pathway had a phone that used iOS? (1 mark)

Solution
27%
Specific behaviours
✓ percentage

(b) Assuming that the survey results apply to the population in general,

(i) does knowing the operating system of a person's phone help you know the pathway they followed at school? Explain your answer. (2 marks)

Solution
No. There are slight differences for each OS (e.g. for Android users, 58% ATAR v 59% non-ATAR) but none of these differences are significant.
Specific behaviours
✓ indicates no ✓ explanation

(ii) does knowing the pathway a person followed at school help you know the type of operating system their phone has? (1 mark)

Solution
No.
Specific behaviours
✓ indicates no

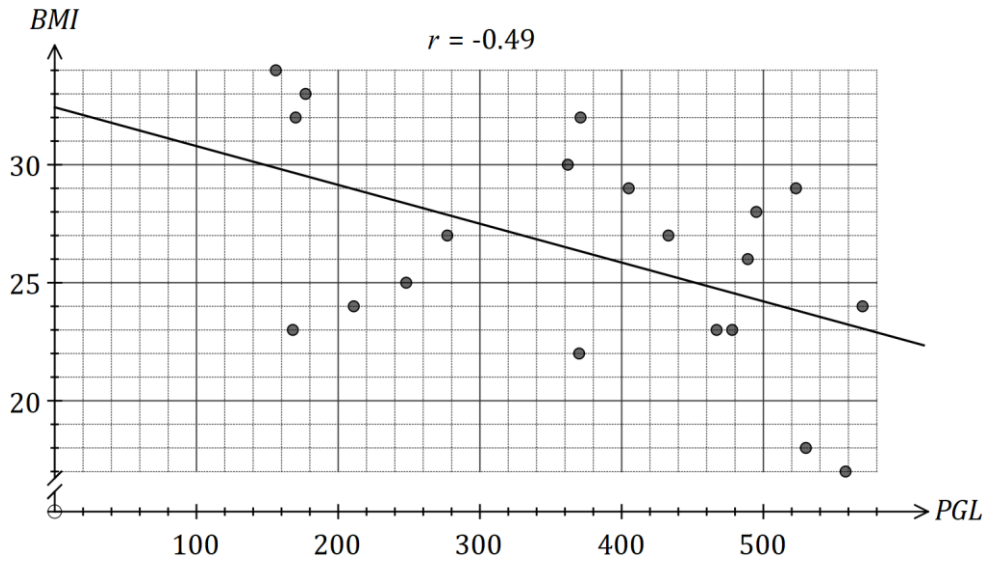
(iii) is there an association between a person's school pathway and type of operating system their phone has? Explain your answer. (2 marks)

Solution
No. Knowing which category a person is in for one variable does not help place the person in a category for the other variable and so there is no association.
Specific behaviours
✓ states no ✓ explanation

See next page

Question 7 **(7 marks)**

A medical study measured the body mass index (BMI) and plasma ghrelin levels (PGL) of a group of patients. The results were displayed in the scatterplot below, together with the least-squares line of best fit and the correlation coefficient between the variables.



(a) How many patients in the study with a PGL of more than 300 had a BMI between 18.5 and 24.5? (1 mark)

Solution
4 patients
Specific behaviours
✓ correct number

(b) Determine the lower and upper predicted BMI for patients with a PGL between 270 and 390. (2 marks)

Solution
BMI between 26 and 28
Specific behaviours
✓ lower bound, ✓ upper bound

(c) Comment on the claim that a high level of plasma ghrelin causes a patient to have a low body mass index. (2 marks)

Solution
The claim is not valid. An observed association does not mean there is a causal relationship between the variables.
Specific behaviours
✓ indicates claim not valid ✓ comments on causality

(d) State the number of patients in the study and comment on how the size of the study could influence any explanation for an association between the variables. (2 marks)

Solution
There were 20 patients in the study. This is small number and increases the chance that any association observed may simply be due to coincidence.
Specific behaviours
✓ number in study ✓ comment linking small samples to unreliable outcomes

See next page

Question 8 **(8 marks)**

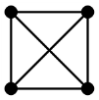
A connected planar graph G_2 has four faces and four vertices.

(a) Determine the number of edges graph G_2 has. (2 marks)

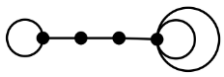
Solution
$e = f + v - 2 \Rightarrow e = 4 + 4 - 2 = 6$
Specific behaviours
✓ indicates use of Euler's formula ✓ edges

(b) In each of the following, use the additional condition only within that part of the question.

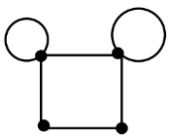
(i) Draw graph G_2 so that it is simple. (2 marks)

Solution
 *(Example solution)*
Specific behaviours
✓ connected planar graph ✓ simple *(no loops or multiple edges)*

(ii) Draw graph G_2 so that it contains a Hamiltonian path but not a Hamiltonian cycle. (2 marks)

Solution
 *(Example solution)*
Specific behaviours
✓ connected planar graph ✓ semi-Hamiltonian (open path thru' all vertices once) *(check for bridge...)*

(iii) Draw graph G_2 so that it contains a Eulerian trail. (2 marks)

Solution
 *(Example solution)*
Specific behaviours
✓ connected planar graph ✓ Eulerian (closed trail using all edges once)

End of questions

Supplementary page

Question number: _____